Name:_____

Summer 2024

AP Calculus BC Summer Work 2024

DIRECTIONS: The following problems are all similar to what was covered in honors precalculus. Since AP Calculus BC picks up where you left off without much review, it is important to have the concepts and problem solving strategies you learned in honors precalculus fresh in your mind. Thus, the following problems are to help motivate you to do that, and they are your summer assignment. Completing this work towards the end of the summer (or at least reviewing your work towards the end of summer if you knock this out early) would be recommended so it is fresh in your mind. If you have forgotten how to do a problem, consider looking at your class notes (keep these from last year!) or reaching out to a classmate for help. Please also either use graph paper (if turning in a paper copy of your work) or use the graph paper option in Notability to help you draw accurate graphs. As always, show work and use proper notation. This summer work is due on the first day of class. You should also expect a test covering this material during the second week of classes. And while some time in class will be dedicated to reviewing this material from prior courses, the expectation is that most of this review should be done independently by you. If needed, I am always happy to help outside of class, too.

Also, it is required that you have a graphing calculator for AP Calculus. I strongly recommend the TI-84 (as this model will be used during in-class demonstrations). If you own another model/brand, please contact me (msheehy@stjohnschs.org) via email to confirm that the calculator can perform the functions required for class and the AP exam.

Lastly, please complete the below problems without a calculator unless otherwise specified.

1. Sketch the graph of the line $2x - 3y = 6$.

2. Sketch the graph of $y = 2x^2 - 1$. State any *x*- or *y*-intercepts. Is this graph symmetric to the *y*-axis, origin or neither?

3. Given: $f(x) = -x^2 - 2x + 3$, Find: (a) $f(-3)$ (b) $f(x+h)$

(c) $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ (d) $f'(x)$ (e) $f'(-3)$

(f) the equation of the tangent line to $f(x)$ at $x = -3$.

4. State the domain and range of the following functions:

(a) $h(x) = \sqrt{25 - x^2}$ (b) $g(x) = \frac{|x-2|}{x-2}$

5. Given $f(x) = \sqrt{5 - x}$ and $g(x) = 3 + 2x$, find the composition $f[g(x)]$.

6. Solve the following equations:

(a) $-2 + x = -6x^{2}$ (b) $x^{3} - x = 0$ (c) $\frac{2x-5}{x-1} = 0$

7. Are the following functions even, odd or neither? (a) $f(x) = 3x^{4} - 5$ (b) $g(x) = \frac{7x}{x^{2}+3}$

8. Evaluate the following without using a calculator:

(a) $sin\left(-\frac{4\pi}{3}\right)$ (b) $cos(45^{\circ})$ (c) $tan\left(\frac{7\pi}{4}\right)$ (d) $sin\left(630^{\circ}\right)$

9. Find the point(s) of intersection of the graphs of $x - y - 1 = 0$ and $y - x^{2} = -7$

10. Write the equation of the line passing the thru the point (2,-7) and having a slope of 2/9.

11. Are the lines $6x + 3y = 15$ and $y = \frac{x}{2} - 3$ parallel, perpendicular or neither? Explain.

12. Find all solutions to the equation $cos\theta csc\theta - 2cos\theta = 0$ in the interval $[0, 2\pi)$.

13. Use your graphing calculator to find all solutions to the equation $3 + cosx = x^{2}$ in the interval $(0, 2\pi)$. Be sure your calculator is in radians mode. Round or truncate to 3 places past the decimal.

14. Solve the system of equations with the help of your graphing calculator: $y = lnx$
Again, round or truncate to 3 places past the decimal. $x + y = 5$

15. Find $\lim_{x \to 5^{-}} \frac{x}{5-x}$.

16. $$f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$$

(a) $\lim_{x \to 1^{-}} f(x) =$____ (b) $\lim_{x \to 1^{+}} f(x) =$____ (c) $\lim_{x \to 1} f(x) =$____ (d) $f(1) =$____

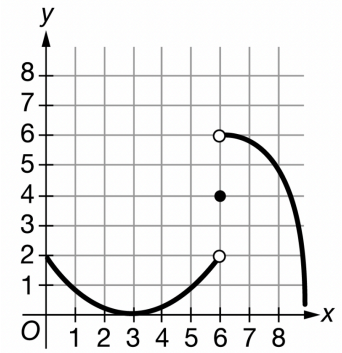
(e) Is $f(x)$ continuous at $x = 1$? Justify your answer with the limit definition of continuity.

17. Write the equations of any vertical asymptotes that the graph of the function $f(x) = \frac{6x^{2}+x-2}{9x^{2}-4}$ would have.

18. MULTIPLE CHOICE. Suppose $f(x)$ is continuous over the closed interval [2,10] and that $f(2) = 19$ and $f(10) = 13$. Then for at least one value of x in the interval [2,10], $f(x)$ must equal what value?

(A) -7 (B) 0 (C) 7 (D) 14 (E) 21

19. Assume all "key points" are integers in answering the below question parts.



Graph of *f*

(a) $\lim_{x \to 6^-} f(x) =$____ (b) $\lim_{x \to 6^+} f(x) =$____ (c) $\lim_{x \to 6} f(x) =$____ (d) $f(6) =$____

20. Find $\frac{dy}{dx}$ if $5x^2y^4 = 2y - sin(3x + 1)$.

21. Find the equation of the tangent line to $g(x) = \frac{10}{3x-4}$ at the point $(2, 5)$.

22. Find $f'(x)$ for the below functions.

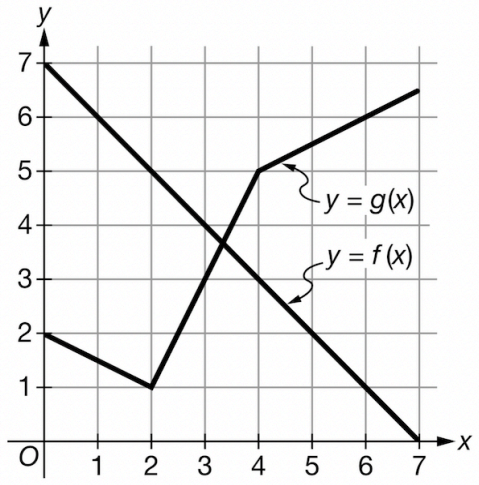
(a) $f(x) = 7\sqrt{6x - 5}$ (b) $f(x) = 7\sqrt{sinx}$ (c) $f(x) = cos(2x)$

(d) $f(x) = cos^7(2x)$ (e) $f(x) = 5e^{2x-3}$ (f) $f(x) = arccos(x^3)$

23. The edges of a cube are increasing 3 centimeter per minute. How fast is cube's volume changing when the volume is 8 cubic centimeters? Label your answer with correct units.

24. Suppose the position of a particle is moving along the *x*-axis at any time $t \geq 0$ is given by the equation $x(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 - 5t + 7$. At what time $t \geq 0$ will the particle be at rest? What is the particle's acceleration at that time?

25. The graphs of the linear function *f* and the piecewise linear function *g* are shown in the graph below. If $h(x) = \frac{f(x)}{g(x)}$, then find $h'(3)$.



26. Find $f'(2)$ if $f(x) = ln\left(\frac{\sqrt{4x+1}}{x-1}\right)$.

27. $f(x) = \sqrt{4x + 1}$ is an increasing function with $f(6) = 5$. Let h be the inverse function of f. Find $h'(5)$.

28. A small swimming pool contains 1000 gallons of water when the pool is uncovered at 8am. Once uncovered, the heat from the sun begins evaporating water from the pool at a constant rate of 20 gallons per hour until the pool closes at 5pm. At noon, a hose is turned on and water is added to the pool at a constant rate of 25 gallons per hour until 2pm. At 2pm, the water entering the pool from the hose is reduced to a constant rate of 10 gallons per hour. The hose is shut off when the pool closes at 5pm.

(a) How many gallons of water are in the pool at 12pm noon?
(b) How many gallons of water are in the pool at 1pm?
(c) Is the water level in the pool rising or falling at 1pm? Explain your answer.
(d) How many gallons of water are in the pool at 4pm?
(e) Is the water level in the pool rising or falling at 4pm? Explain your answer.